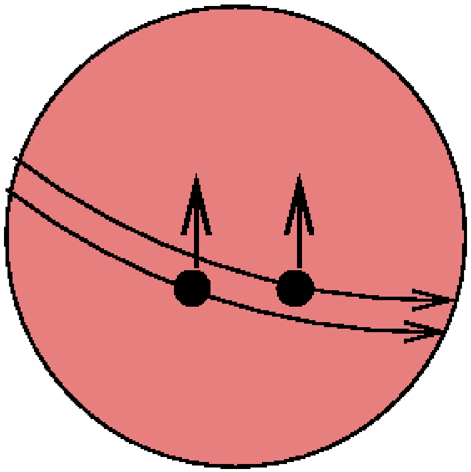
Nuclear Structure from Gamma-Ray Spectroscopy

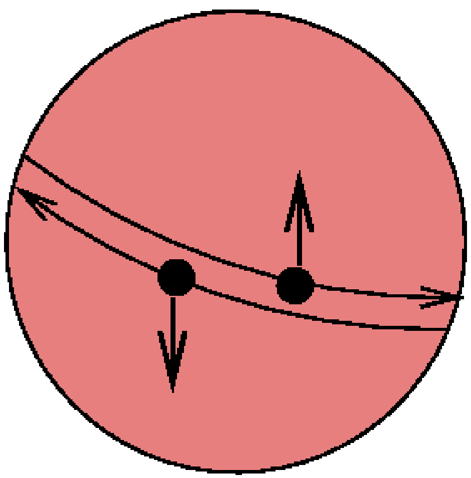
2019 Postgraduate Lectures

Lecture 2: Low-spin nuclear structure

Time Reversed Orbits



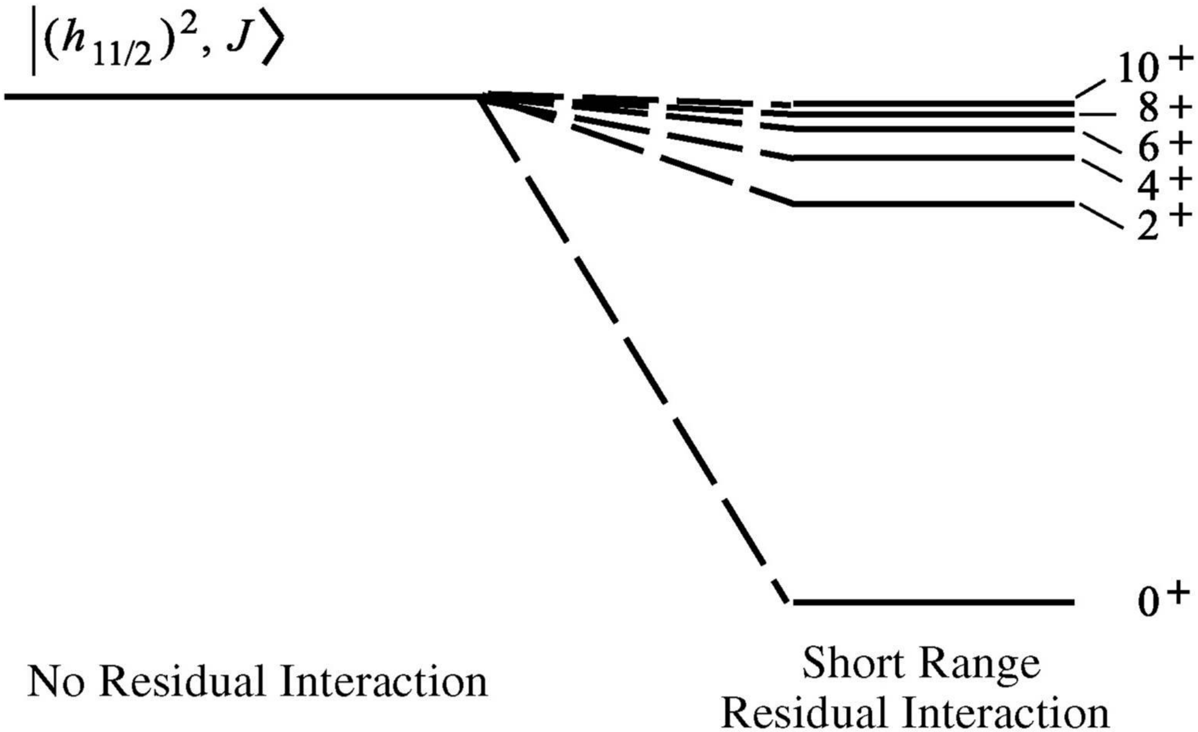
Pauli forbidden



Time reversed: two interactions per revolution

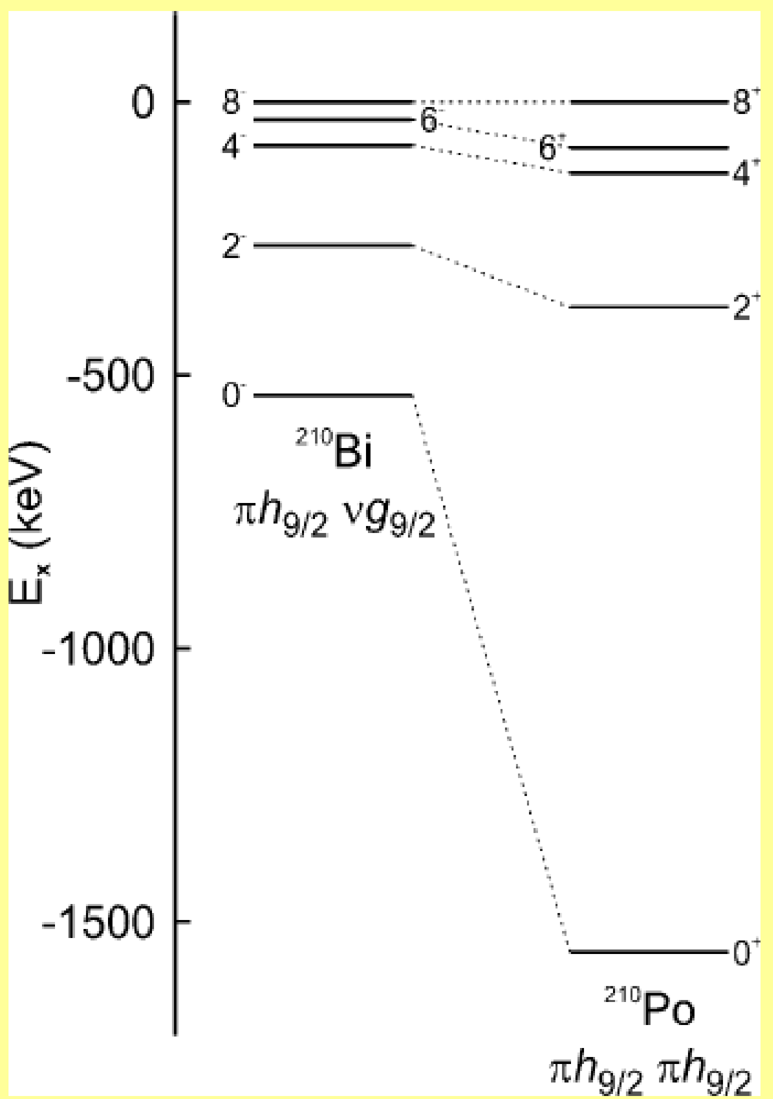
- The greatest overlap would occur if two particles could orbit in the same level
- Not allowed (PEP) !
- The next greatest overlap occurs for particles in 'time reversed' orbits
- The spins cancel to give $I^{\pi} = 0^{+}$

Coupling Two Particles



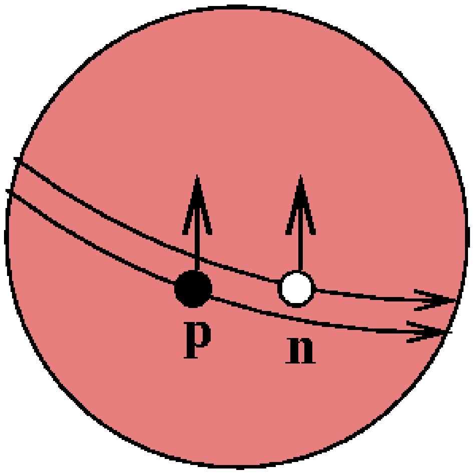
- The short-range (pairing) residual interaction yields an energetically favoured 0^+ state

Favoured 0^+ Ground State



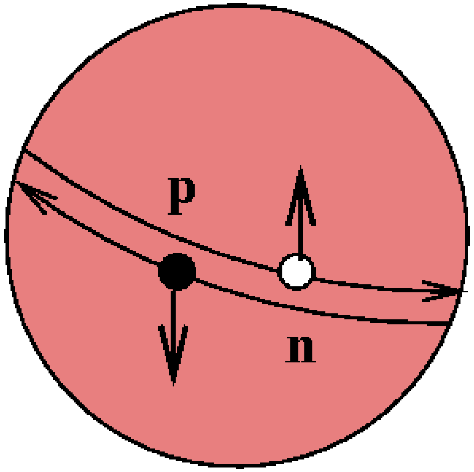
- In ^{210}Po the configuration outside the doubly closed shell core of ^{208}Pb is $(\pi h_{9/2})^2$.
- If there were no interaction between these two protons, i.e. if they behaved like independent particles, the various $(h_{9/2})^2$ spin couplings, which reflect the orbital alignments, would lead to states degenerate in energy.
- Correlated pair of two protons
- Energy gain ≈ 2Δ

Neutron-Proton Pairing



Spin I = 1

Isospin T = 0

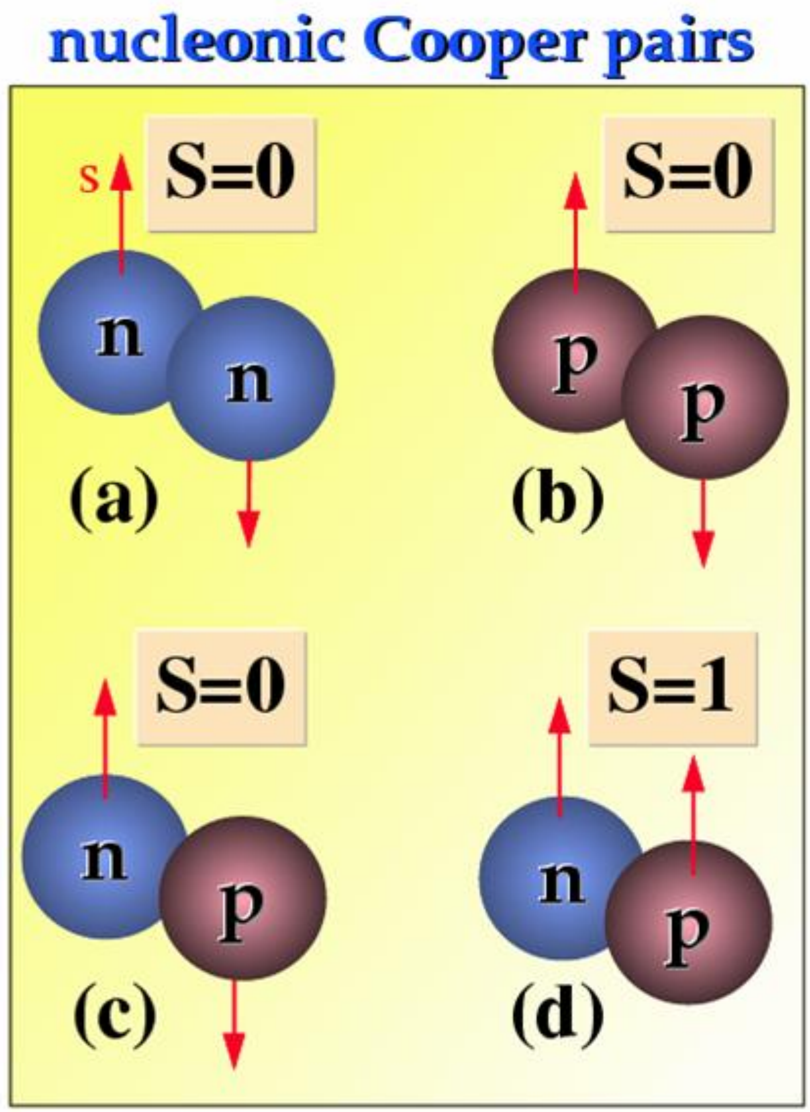


Spin I = 0

Isospin T = 0

- The concept of superconductivity, related to like nucleon pairs coupled to spin I = 0 and isospin T = 1, can be extended to neutron-proton pairs with T = 0
- The greatest overlap occurs if the particles are in the same orbitals
- Strong neutron-proton pairing can occur for nuclei with N = Z

Nucleon Pairing



- The isovector (T=1) n-p pairing (c) is similar to the n-n (a) and p-p (b) pairing
- The isoscalar (T=0) n-p pairing (d) is clearly different

Moment of Inertia

- Deformation provides an element of anisotropy allowing the definition of a nuclear orientation and the possibility of observing rotation

- Classically the energy associated with rotation is:

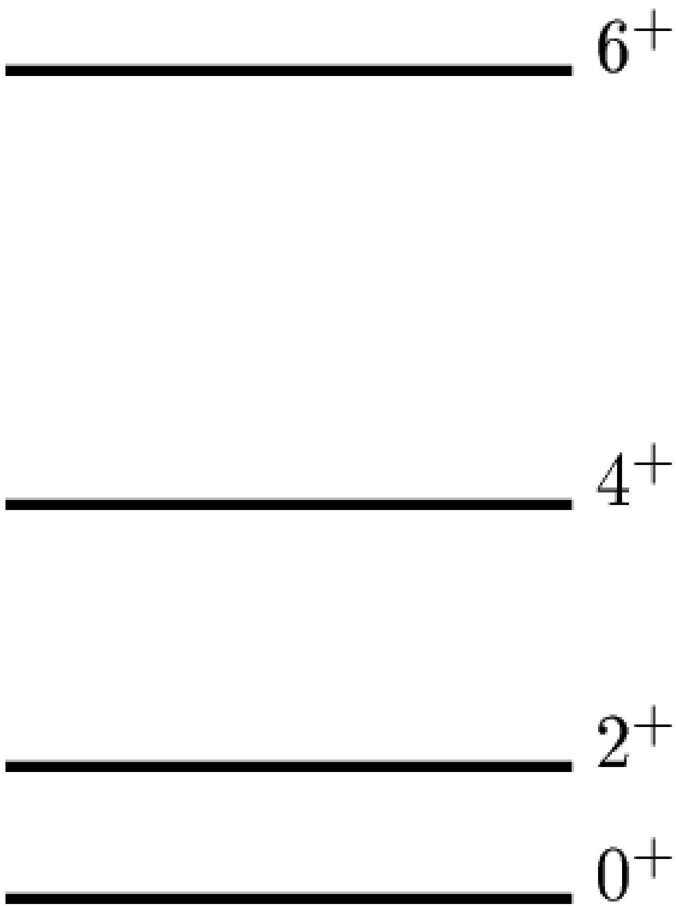
$$E_{rot} = \tfrac{1}{2} \Im \omega^2 = I^2 / 2 \Im \; ; \; \omega = I / \Im$$

- Collective rotation involves the coherent contributions from many nucleons and gives rise to a smooth relation between energy and spin:

$$E = (\hbar^2/2\Im)\, I[I + 1]$$

which defines the 'static' moment of inertia, sometimes denoted $\Im^{(0)}$

Energy Levels of a Rotor

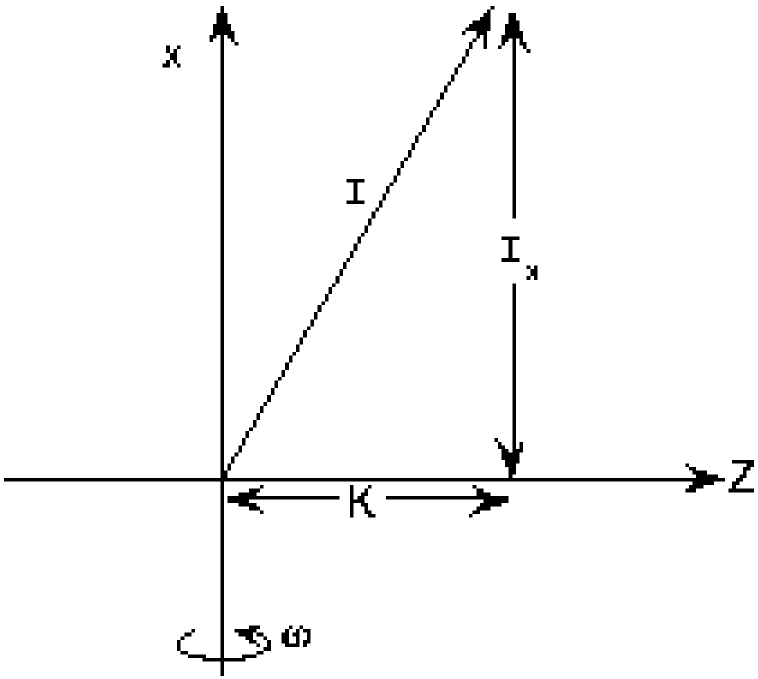


- The energy levels of a rotor are proportional to I(I+1)
- The ratios of energy levels for a rotor are:

$E(4^+)/E(2^+) = 3.333$

$E(6^+)/E(2^+) = 7.0$

Rotational Frequency



- The intensive variable ω (rotation about the x axis) is related to the extensive variable I by the relation:

$$\hbar\omega = dE/dI_x$$
$$\approx \tfrac{1}{2}[E(I+1) - E(I-1)]$$

- Here I_x is the projection of I onto the rotation axis (x):

$$I_x = \sqrt{[I(I+1)-K^2]}\,\hbar$$

The rotational frequency ω is distinct from the oscillator quantum ω_0. In practice $\omega \ll \omega_0$ and the collective rotation can be considered as an adiabatic motion

Rigid Body Moment of Inertia

- The rigid-body moment of inertia for a spherical nucleus is:

$$\mathfrak{I}_{rig} = (2/5)\, MR^2 = (2/5)\, A^{5/3}\, m_N\, r_0^2$$

where m_N is the mass of a nucleon ($M = A\, m_N$) and

$$R = r_0\, A^{1/3} \quad \text{with} \quad r_0 = 1.2 \text{ fm}$$

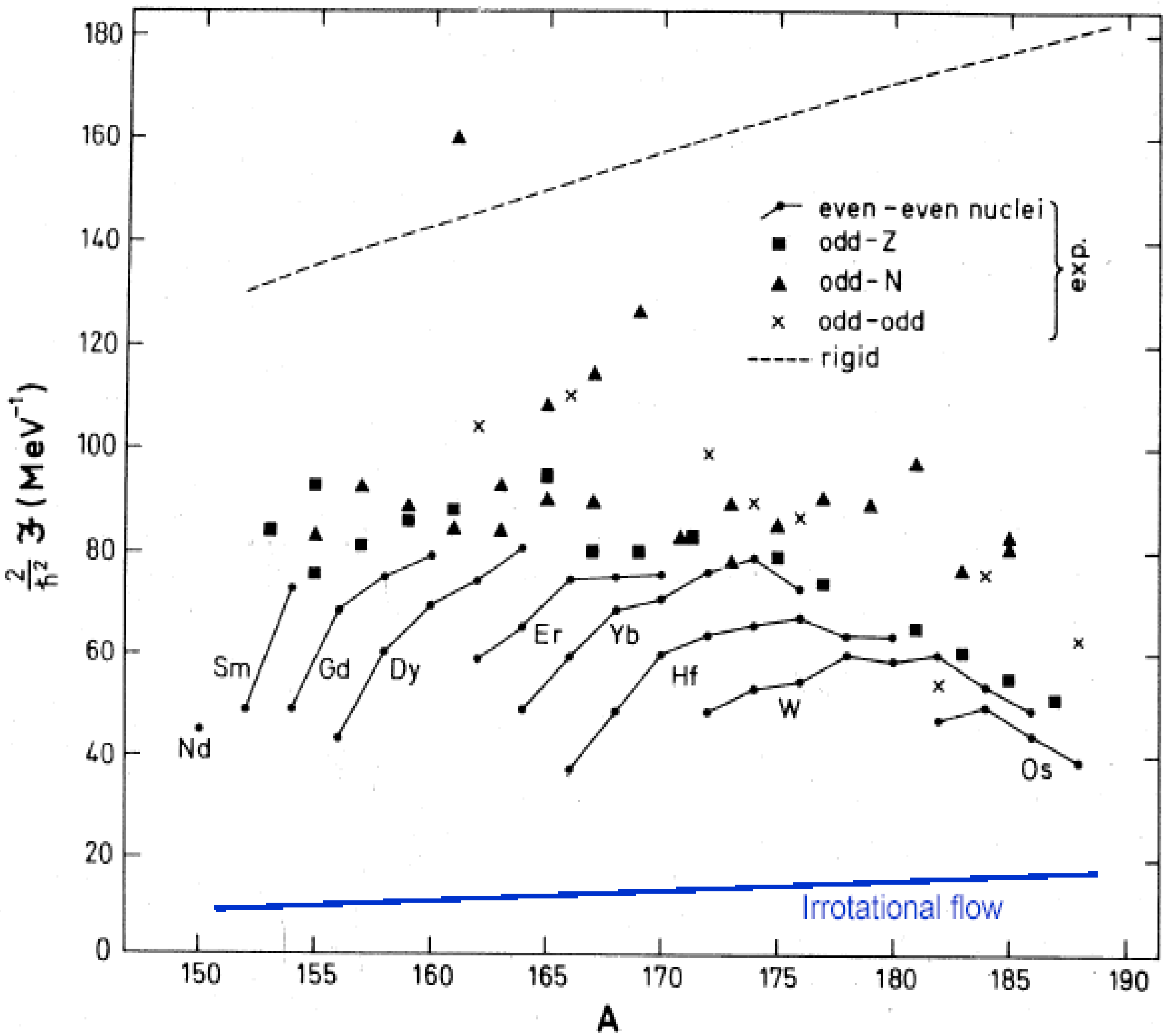
- For a deformed nucleus:

$$\mathfrak{I}_{rig} = (2/5)\, A^{5/3}\, m_N\, r_0^2\, [1 + 1/3\, \delta]$$

where $\delta = \Delta R / R_0$

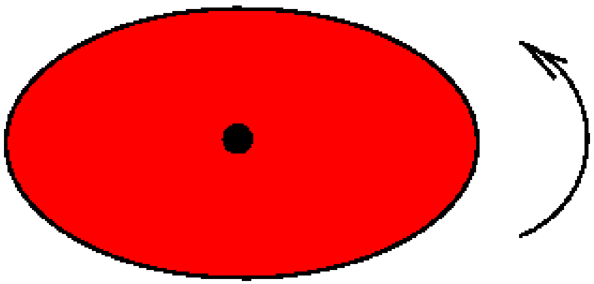
- Typically nuclear moments of inertia are less than 50% of the rigid-body value at low spin

Nuclear Moments of Inertia

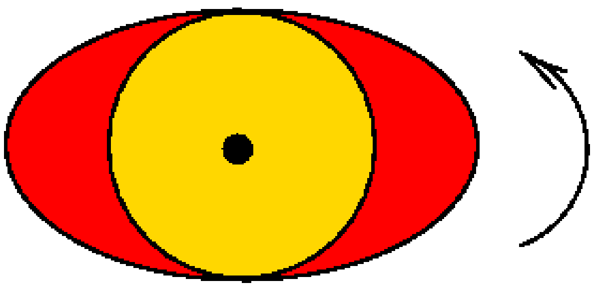


- Nuclear moments of inertia are lower than the rigid-body value – a consequence of nuclear pairing

Nuclear Rotation



Rigid body



Nucleus

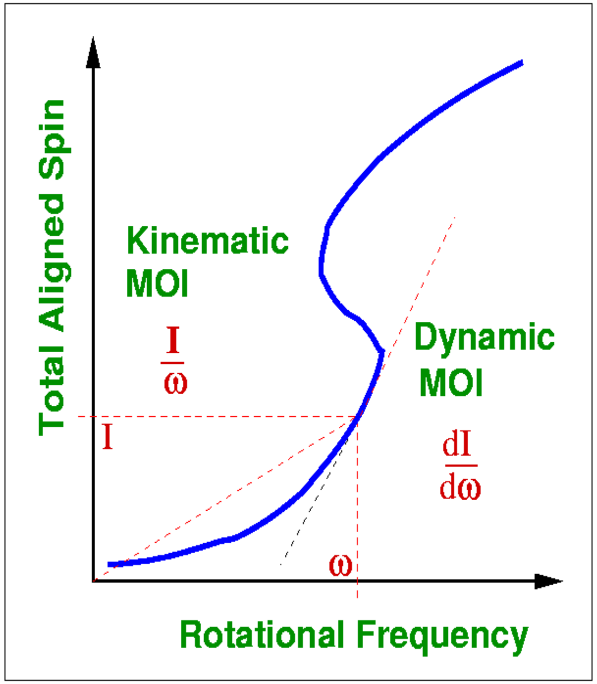
- The assumption of the ideal flow of an incompressible nonviscous fluid (Liquid Drop Model) leads to a hydrodynamic moment of inertia (surface waves):

$$\Im_{hydro} = \Im_{rig}\,\delta^2$$

- This estimate is much too low !

- We require short-range <u>pairing</u> correlations to account for the experimental values

Kinematic and Dynamic MoI's



- Assuming maximum alignment on the x-axis ($I_x \sim I$), the kinematic moment of inertia is defined:

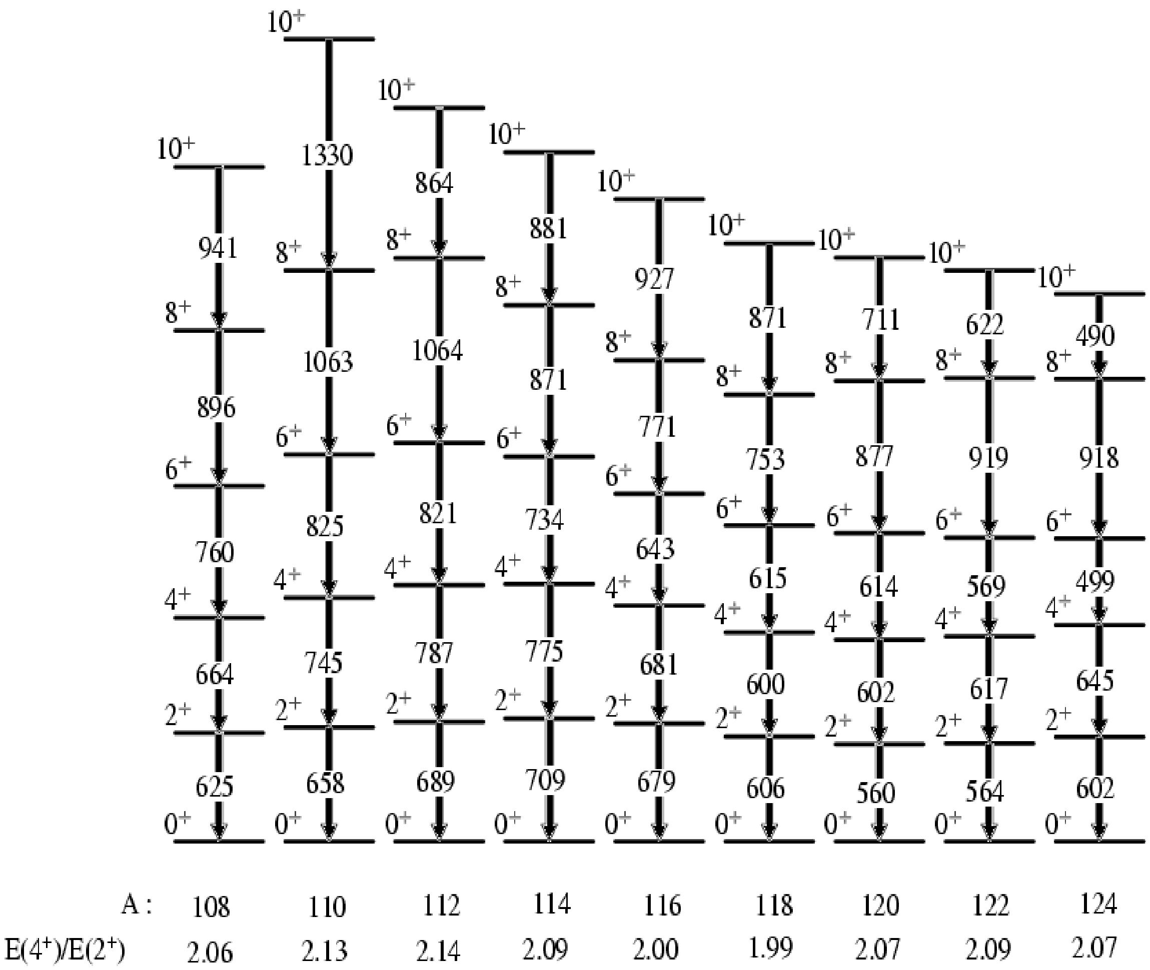
$$\Im^{(1)} = (\hbar^2 I)\,[dE(I)/dI]^{-1} = \hbar\, I/\omega$$

- The dynamic moment of inertia (response of system to a force) is:

$$\Im^{(2)} = (\hbar^2)\,[d^2E(I)/dI^2]^{-1} = \hbar\, dI/d\omega$$

- Note that $\Im^{(2)} = \Im^{(1)} + \omega\, d\Im^{(1)}/d\omega$
- Rigid body: $\Im^{(1)} = \Im^{(2)}$ Nucleus at high spin: $\Im^{(1)} \approx \Im^{(2)}$

Vibration or Rotation?



- The simple ratio of the 4^+ and 2^+ energy levels of an even-even nucleus gives an indication of the types of excitation
- For a vibrator:
  - $E \propto n$
  - $E(4^+)/E(2^+) = 2.0$
- For a rotor:
  - $E \propto I(I+1)$
  - $E(4^+)/E(2^+) = 3.33$

Te (Z = 52) systematics show that they are vibrational

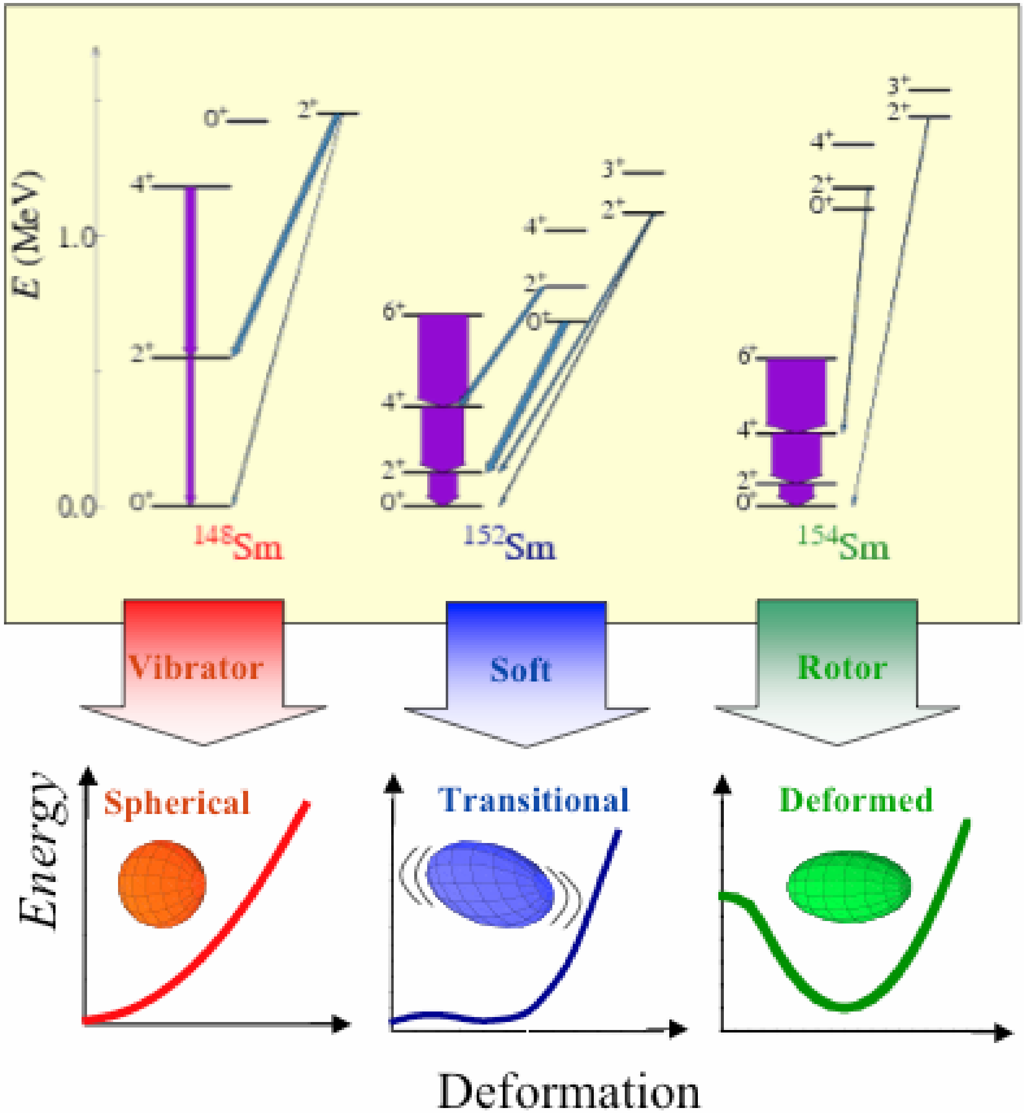
Development of Collectivity

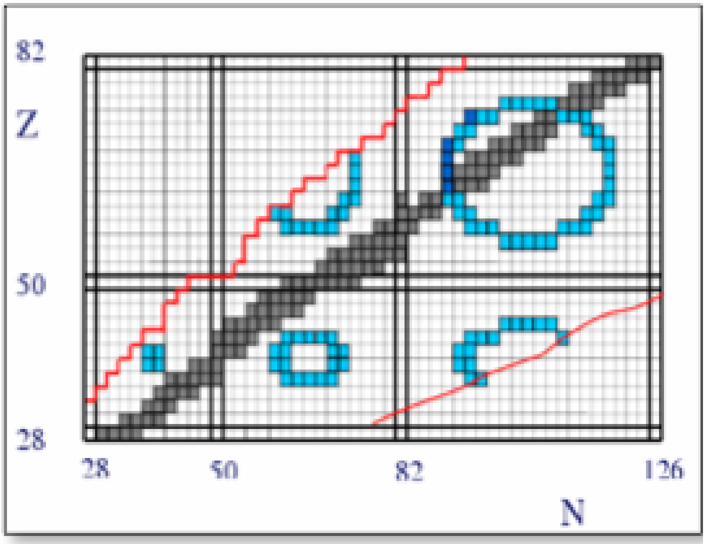
- Another limiting value of the $E(4^+)/E(2^+)$ ratio is 2.5, corresponding to a γ-soft rotor, or γ-unstable oscillator (O(6) limit of the interacting boson model: IBM)

- Adding protons to tin:

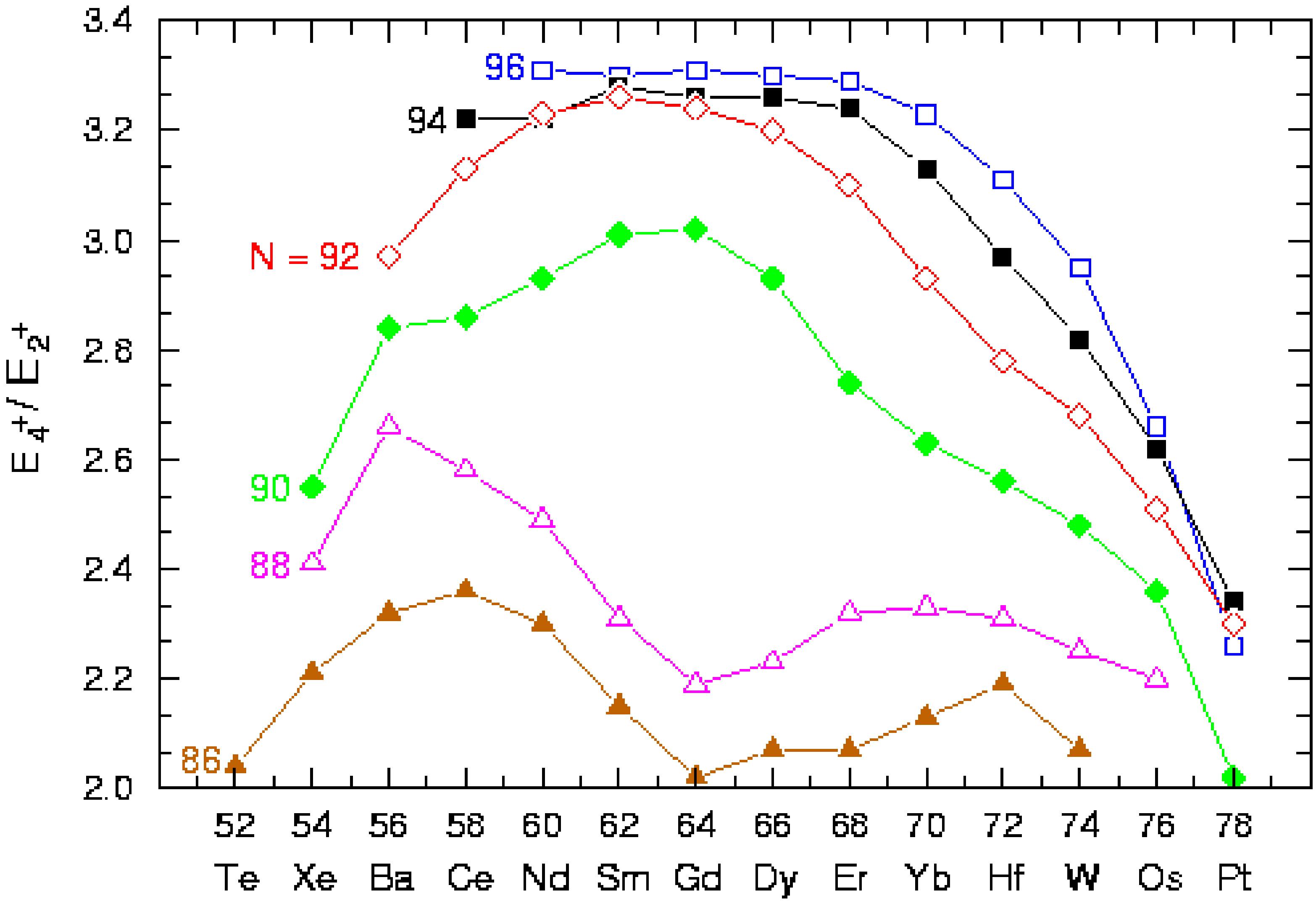
Nucleus	$E(4^+)/E(2^+)$	Behaviour
^{116}Sn (Z=50)	1.65	spherical
^{118}Te (Z=52)	1.99	vibrational
^{120}Xe (Z=54)	2.47	γ-soft
^{122}Ba (Z=56)	2.89	transitional
^{124}Ce (Z=58)	3.15	rotational

Transitional Nuclei

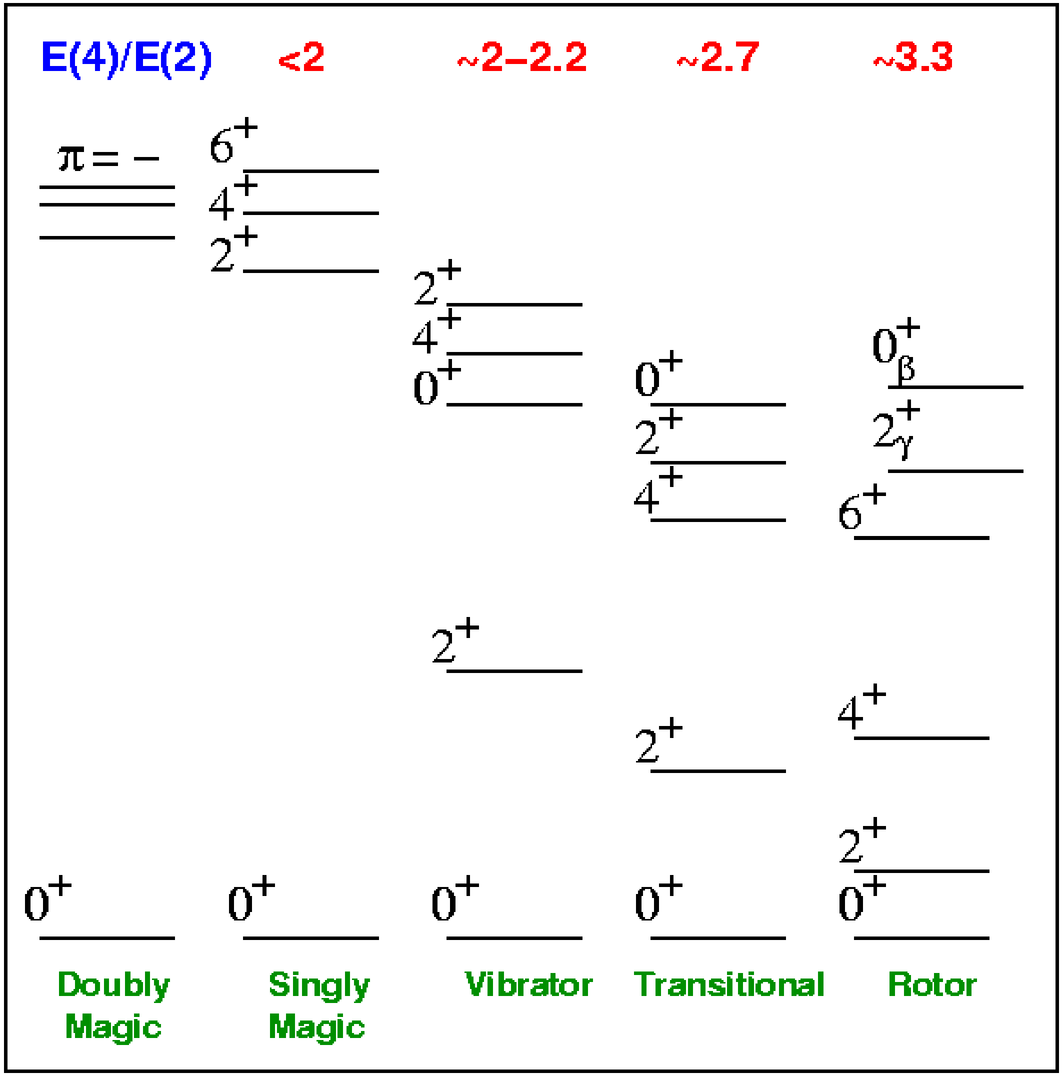




E(4⁺)/E(2⁺) Values


3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
E_{4^+}/E_{2^+}
96
94
N = 92
90
88
86
52 54 56 58 60 62 64 66 68 70 72 74 76 78
Te Xe Ba Ce Nd Sm Gd Dy Er Yb Hf W Os Pt

Evolution of Structure



- This diagram shows the evolution of level structure from closed shell (doubly magic, spherical) to midshell (rotational, deformed) nuclei
- The corresponding $E(4^+)/E(2^+)$ ratios are also shown

Particle-Vibration Coupling

- For an odd-A nucleus near a closed shell with small deformation, the odd particle may couple to the surface vibrations of the core

- The Hamiltonian is: $H = [H_{int} + H_{vib}] + H_{coup} = H_0 + H_{coup}$
- If we assume the interaction $H_{coup} \rightarrow 0$, the motions are decoupled from one another and the eigenfunctions will take a product form: $H_0\Psi = E\Psi$ with $\Psi = \Psi_{int}\,\Psi_{vib}$
- Consider coupling an $h_{9/2}$ proton to the 3^- state in ^{208}Pb, forming states in ^{209}Bi. Seven 'degenerate' states are formed by coupling spin vectors 3 and 9/2

Rotation-Vibration Model

- The RVM model considers a well deformed (static), axially symmetric even-even nucleus and allows small fluctuations (dynamic) about the equilibrium shape β_0
- After a 'few' approximations the energy spectrum may be written as:

$$E = \tfrac{1}{2}\epsilon_R\,[I(I+1) - K^2] + \epsilon_\beta n_\beta + \epsilon_\gamma n_\gamma$$

where the ϵ parameters are energies associated with rotations and vibrations

- ϵ_R is related to β_0 and the nuclear moment of inertia $\mathfrak{I}$

RVM Quantum Numbers

- The quantum numbers are constrained such that:

$K = 0, 2, 4, \ldots$

$I = 0, 2, 4, \ldots$ for $K = 0$

$ = K, K+1, K+2, \ldots$ for $K \neq 0$

$n_\beta = 0, 1, 2, \ldots$

$n_\gamma = K/2, K/2 + 2, K/2 + 4, \ldots$

- So what are the possible low-lying energy levels ?

RVM Band Structure

- For $K = n_\beta = n_\gamma = 0$, we expect a set of levels:

  $$E = \tfrac{1}{2}\epsilon_R\, I(I+1)$$

  with $I = 0, 2, 4,\ldots$ 'ground-state band' represents pure rotation

- A rotational band can be built on a β vibration by setting $n_\beta = 1$. The energy levels are:

  $$E = \epsilon_\beta + \tfrac{1}{2}\epsilon_R\, I(I+1)$$

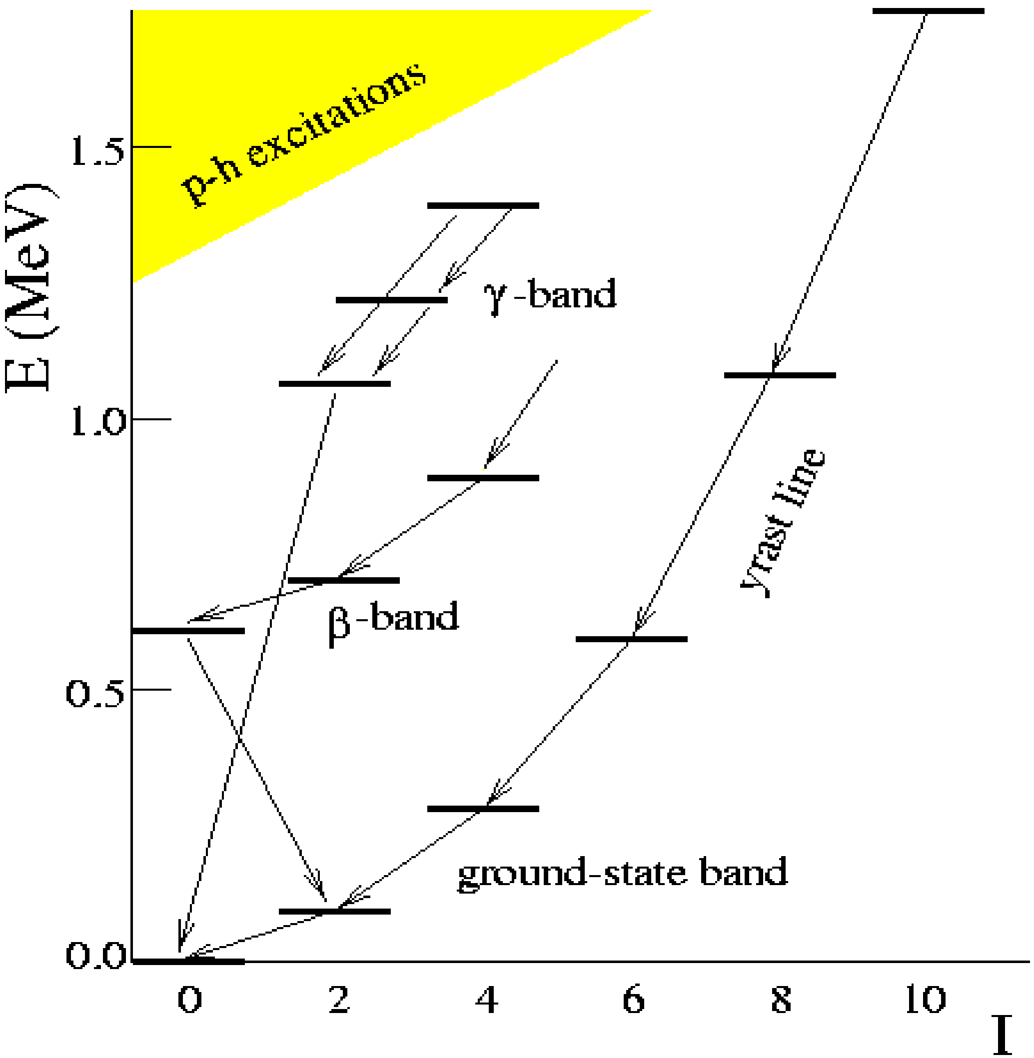
  again with $I = 0, 2, 4,\ldots$ 'β band' (α_{20} varies)

- To include a γ vibration requires a nonzero K. So beginning with $K = 2$ and $n_\gamma = 1$, the levels are:

  $$E = \epsilon_\gamma + \tfrac{1}{2}\epsilon_R\, [I(I+1) - 4]$$

  this time with $I = 2, 3, 4,\ldots$ 'γ band' (α_{22} varies)

β and γ Bands



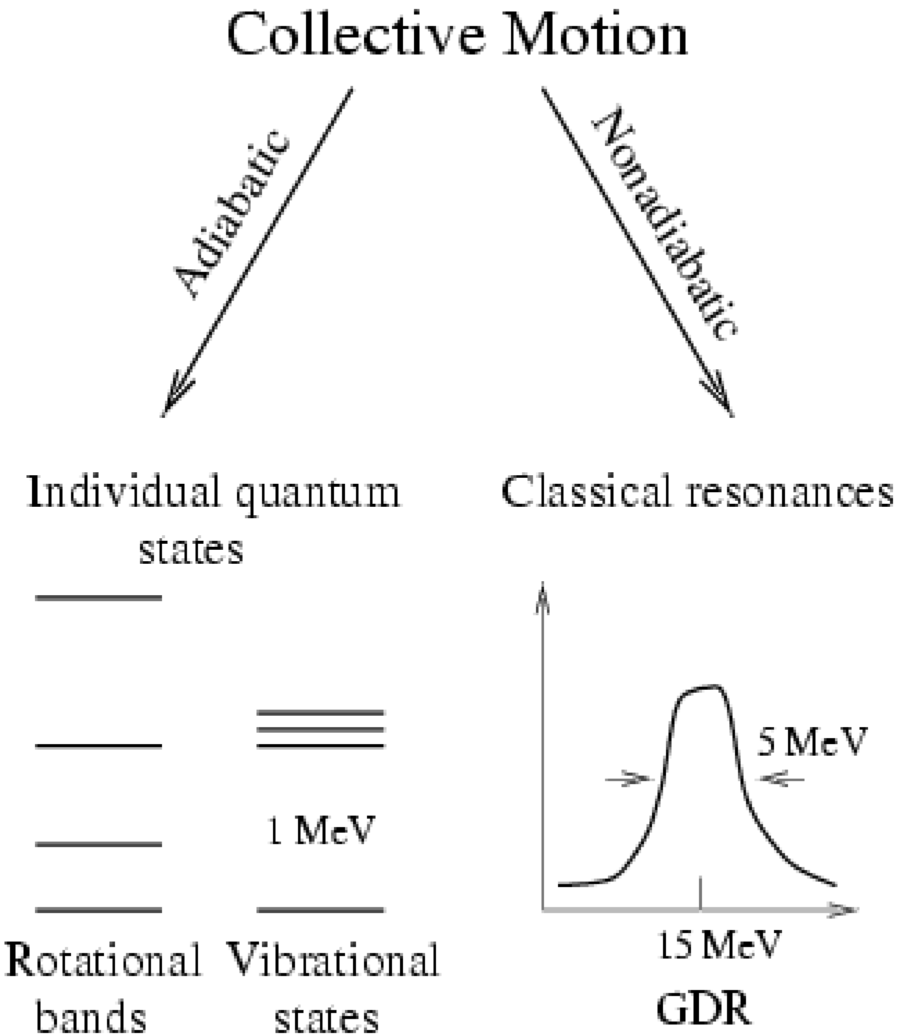
- β-vibrational and γ-vibrational bands coexist with the rotational ground-state band in deformed nuclei
- Such bands are found predominantly in the regions:

  150 ≤ *A* ≤ 190 and

  A ≥ 230

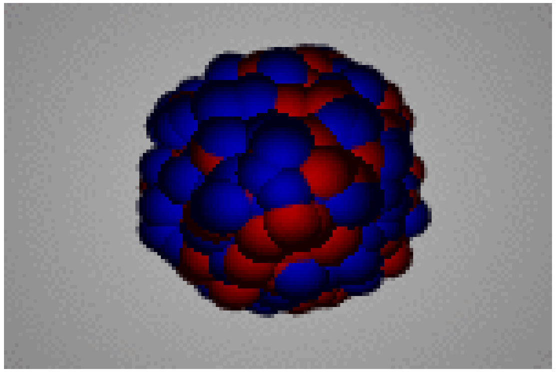
  which are far from shell closures

Nonadiabatic Vibration

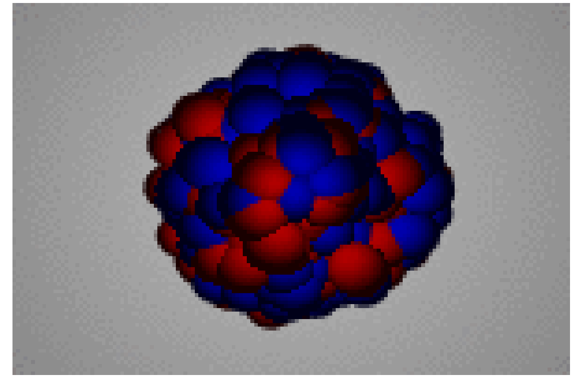


- For the surface modes of vibration, the frequency (velocity) of the oscillations is much smaller than that of the individual nucleonic motion
- The motion is 'adiabatic' (as is nuclear rotation) and individual quantum levels are evident
- However, 'nonadiabatic' collective motion can occur: 'giant resonances'

Giant Resonances



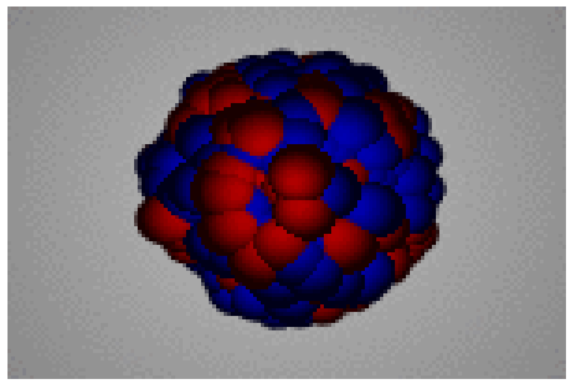
Monopole
L = 0

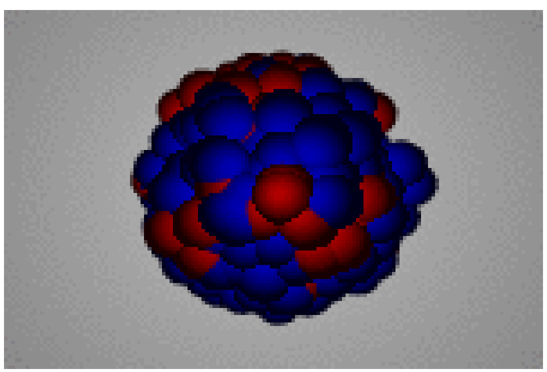


Isoscalar

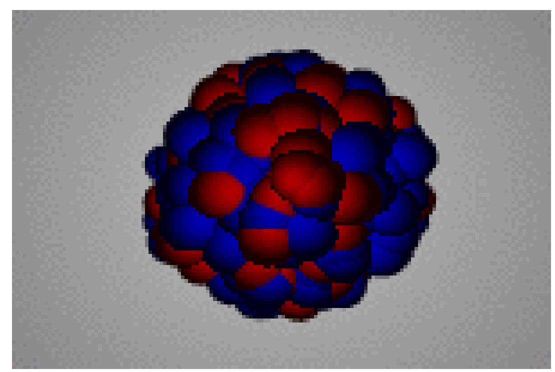
Isovector

Dipole
L = 1





Quadrupole
L = 2



Reflection Asymmetry

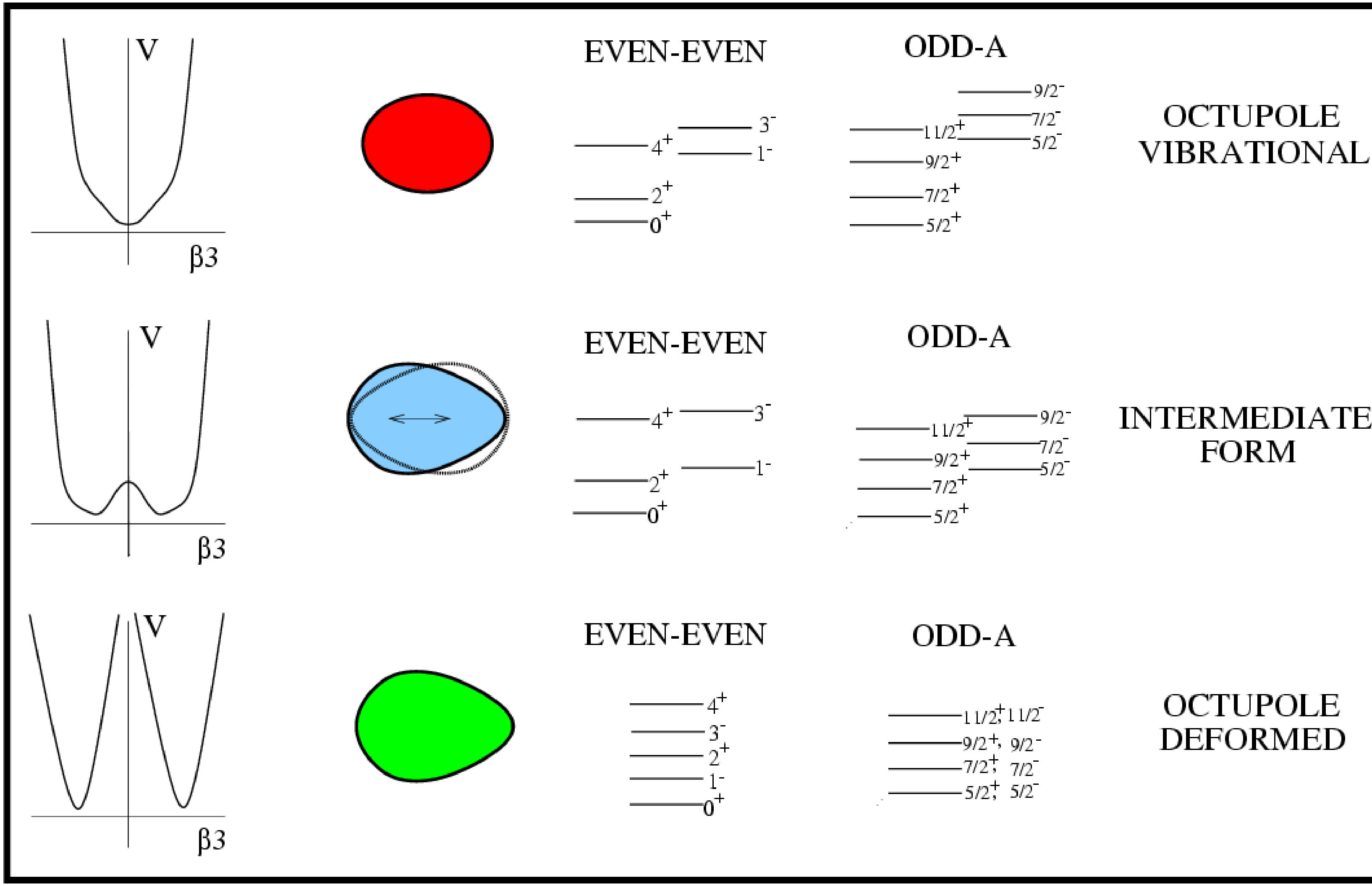
- If a nucleus is 'reflection asymmetric' (i.e. the odd multipole deformation parameters are non-zero, e.g. $\beta_3 \neq 0$ is the most important) then the nuclear wavefunction in its intrinsic frame is not an eigenvalue of the parity operator:

$$\Psi^2(x, y, z) \neq \Psi^2(-x, -y, -z)$$

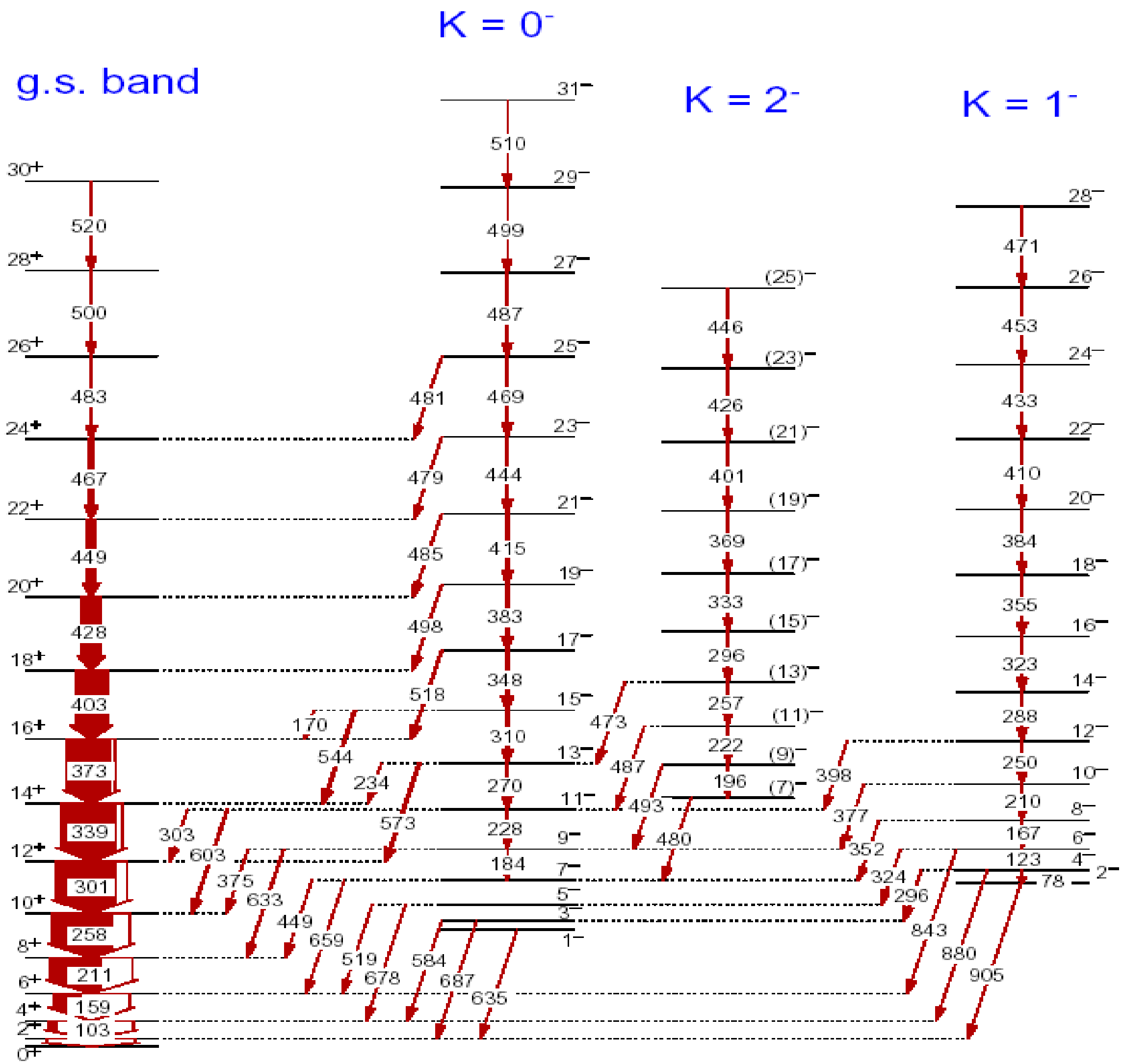
- If $\beta_3 \neq 0$ for a nucleus it is said to possess octupole deformation

- The deformation can however be static, $\langle \beta_3 \rangle \neq 0$, or dynamic, $\langle \beta_3 \rangle = 0$ (oscillating octopule shape)

Octupole Band Structures



Octupole Vibrations in ^{238}U



- This nucleus shows three octupole vibrational bands with different K values

Parity Splitting

- For a static octupole shape, the negative parity states are interleaved (midway between) with the positive parity states

- A measure of such a feature is the 'parity splitting', defined as:

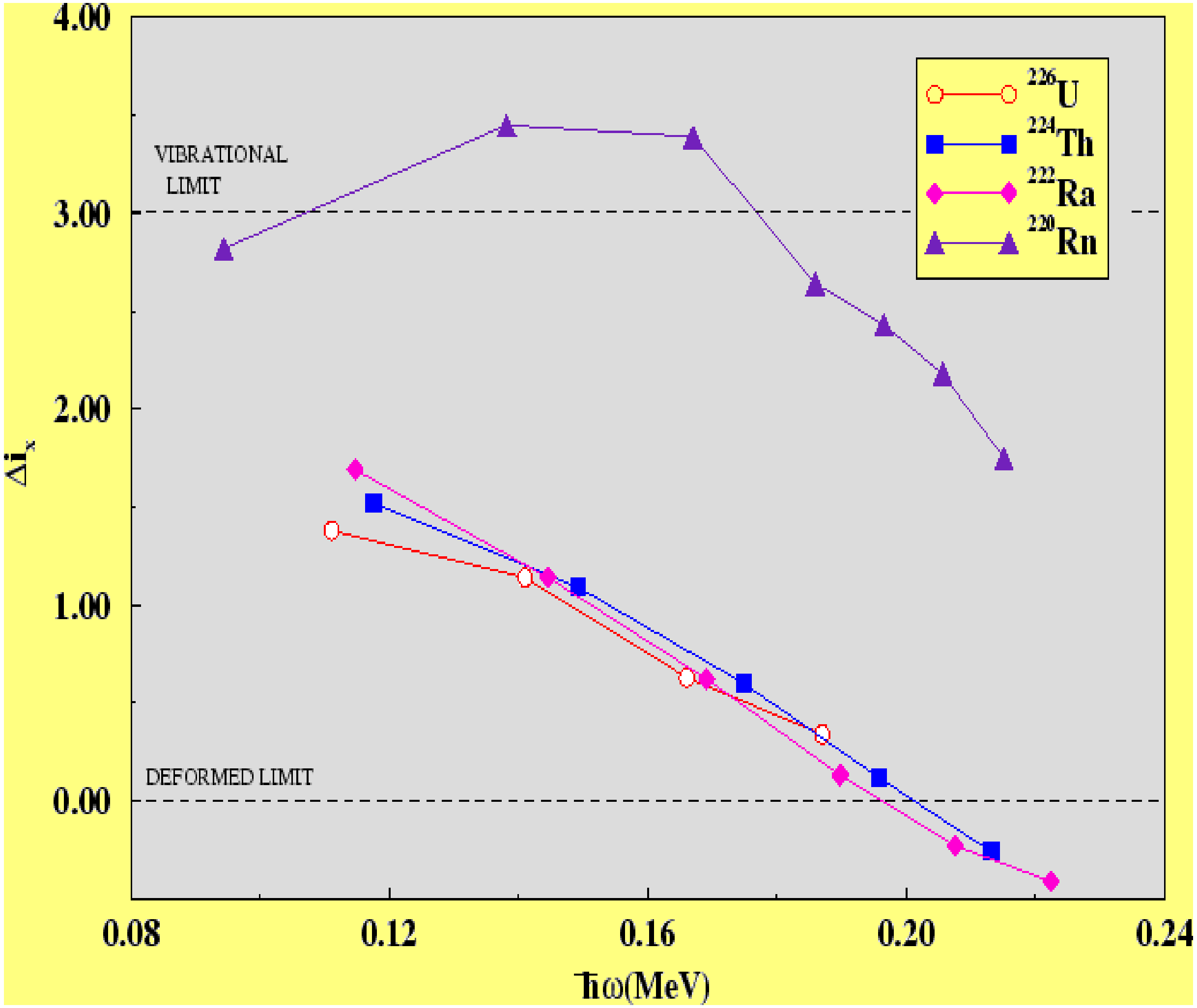
$$\delta E = E(I)^{-} - \tfrac{1}{2}[\, E(I+1)^{+} + E(I-1)^{+} \,]$$

- This quantity generally decreases towards zero with increasing spin and suggests that rotation may stabilise the octupole shape

- A similar quantity is the difference in alignment:

$$\Delta i_x = i_x^{-} - i_x^{+}$$

Octupole Vibration or Deformed?



- For an octupole vibrational phonon coupled to the positive-parity states:

$$\Delta i_x = 3\,\hbar$$

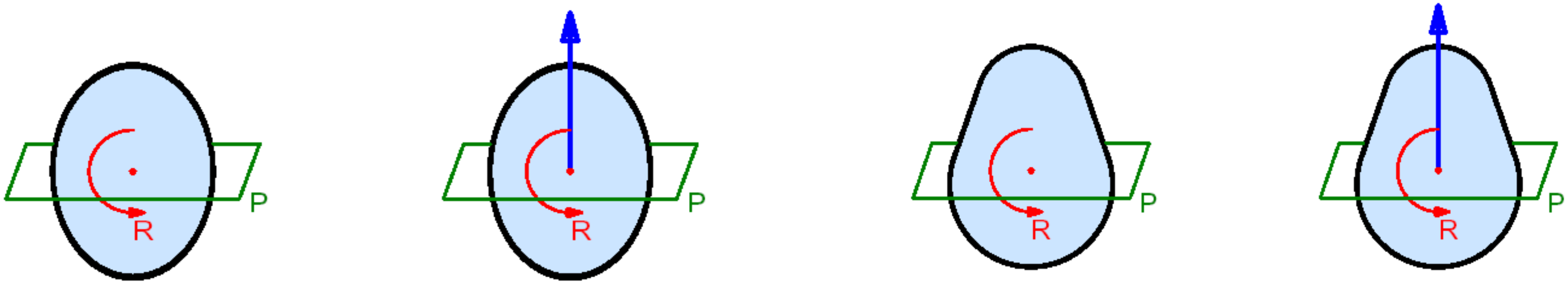
- For a static octupole deformation:

$$\Delta i_x = 0$$

Reflection (A)symmetry

K = angular momentum projection
on symmetry axis

K = angular momentum projection
on symmetry axis



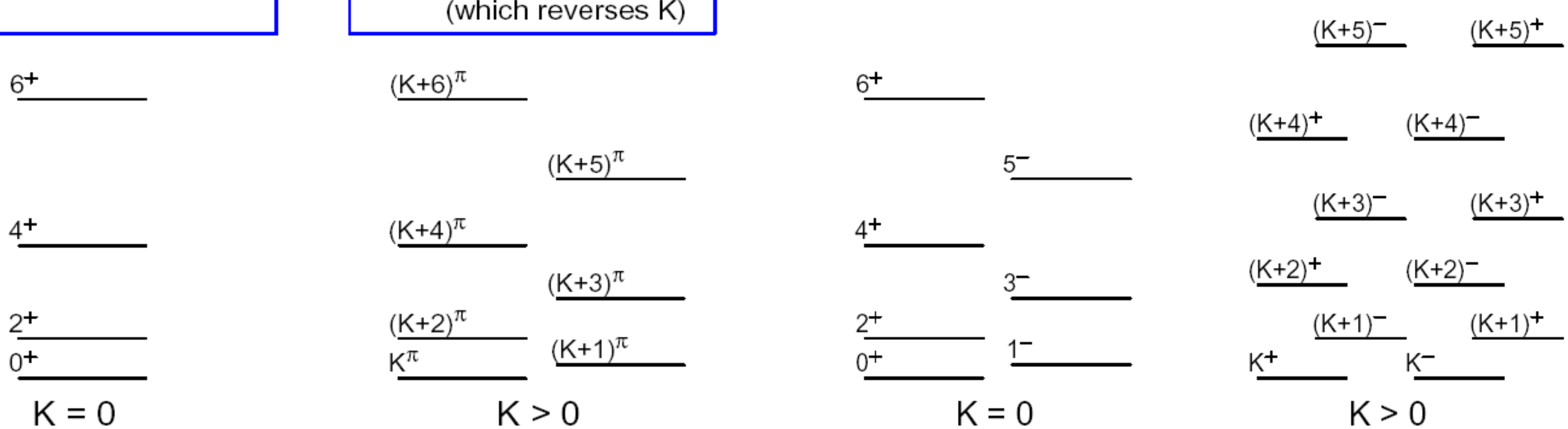
K = 0 | **K > 0** | **K = 0** | **K > 0**

P: parity (reflection)
R: rotation by 180°
T: time reversal

P: parity (reflection)
RT: rotation by 180°
AND time reversal
(which reverses K)

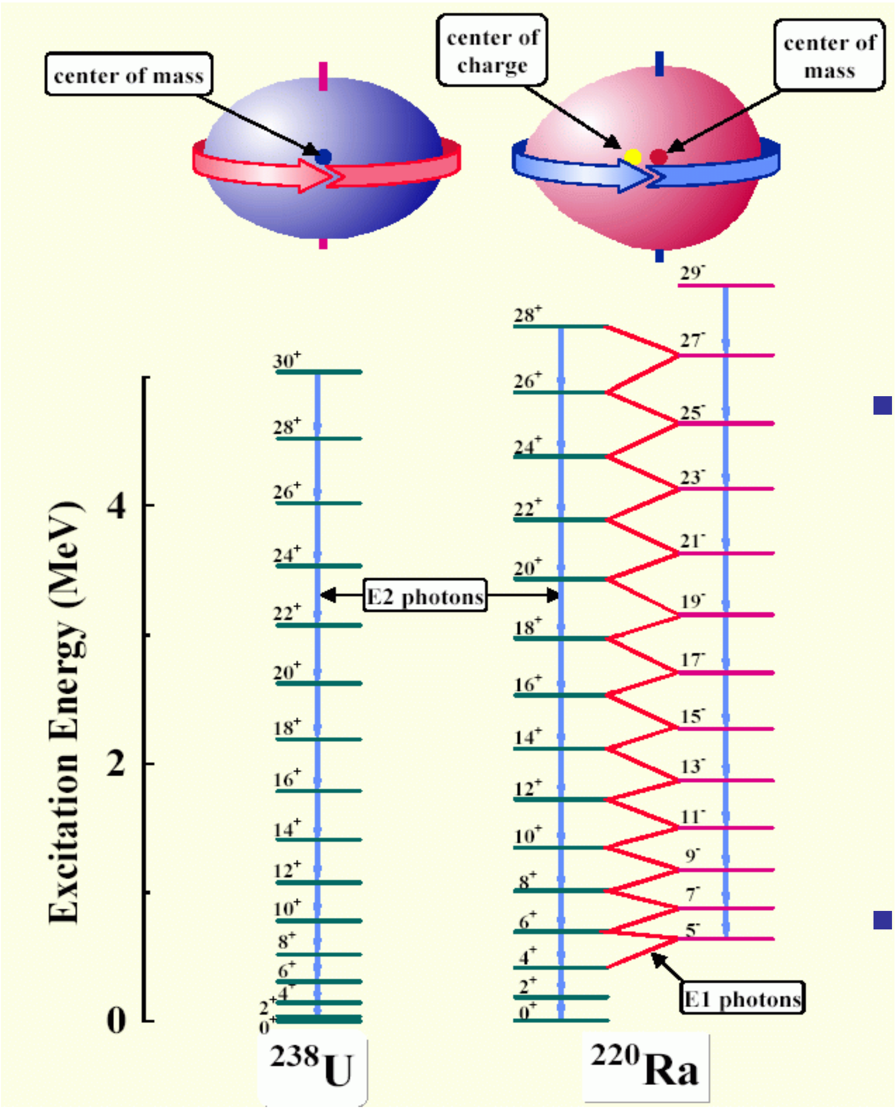
RP: rotation & reflection
T: time reversal

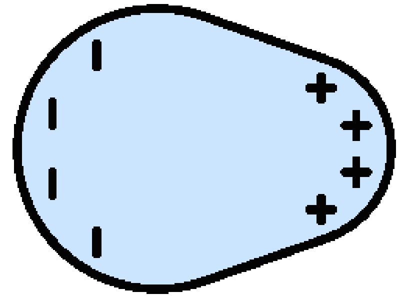
RPT: need all three
operations



1 band | 2 bands | 2 bands | 4 bands

Electric Dipole Moment





- In a nucleus with octupole deformation, the centre of mass and centre of charge tend to separate, creating a non-zero electric dipole moment
- Bands of opposite parity connected by strong E1 transitions occur

Enhanced E1 Transitions

- In heavy nuclei, E1 strengths typically lie between 10^{-4} and 10^{-7} Wu
- In nuclei with octupole deformation, the E1 strengths can be much higher: 10^{-3} – 10^{-2} Wu
- The intrinsic dipole moment of an octupole deformed nucleus is:

$$D_0 = C_{LD} \, A \, Z \, e \, \beta_2 \, \beta_3$$

with the liquid drop constant C_{LD} = 0.0007 fm

- In a Strutinsky type approach, macroscopic and microscopic effects can be considered and:

$$D = D_{macro} + D_{shell}$$

Experimental Dipole Moments

- Experimental values of D_0 can be obtained by measuring B(E1)/B(E2) ratios, related simply to γ-ray energies and intensities

- The B(E1) reduced transition rate is:

$$B(E1; I \rightarrow I-1) = (3/4\pi)\, e^2 D_0^2\, |\langle I_i K_i 1 0 | I_f K_f \rangle|^2$$

- The B(E2) reduced transition rate is:

$$B(E2; I \rightarrow I-2) = (5/16\pi)\, e^2 Q_0^2\, |\langle I_i K_i 2 0 | I_f K_f \rangle|^2$$

- Hence if Q_0 is known (e.g. from the quadrupole deformation β_2) then a value for D_0 can be extracted, i.e:

$$D_0 = \sqrt{[5B(E1)/16B(E2)]}\, Q_0$$